

Topics : Method of Differentiation, Continuity & Derivability, Limits, Solution of Triangle

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.5,6,7,8	(4 marks, 5 min.)	[16, 20]

1. The number of points where $f(x) = [\sin x + \cos x]$, where $[.]$ denotes the greatest integer function, $x \in (0, 2\pi)$ is not continuous is :
 (A) 3 (B) 4 (C) 5 (D) 6

2. $\lim_{x \rightarrow 0} \frac{\sin[\cos x]}{1+[\cos x]}$ ($[.]$ denotes the greatest integer function) is equal to
 (A) equal to 1 (B) equal to 0 (C) does not exist (D) none of these

3. If $x = \cos \theta, y = \sin^3 \theta$, then $\left(\frac{dy}{dx}\right)^2 + y \frac{d^2y}{dx^2} + 3 \Big|_{\theta=\pi/3}$ is equal to
 (A) 0 (B) 1 (C) $\frac{16}{57}$ (D) $\frac{57}{16}$

4. If $4a^2 + c^2 = b^2 - 4ac$, then the variable line $ax + by + c = 0$ always passes through two fixed points. The coordinates of the fixed points can be
 (A) $(-2, -1)$ (B) $(2, -1)$ (C) $(-2, 1)$ (D) $(2, 1)$

5. Let $f(x) = x^3 - 9x^2 + 15x + 6$ and $g(x) = \begin{cases} \min f(t); & 0 \leq t \leq x, 0 \leq x \leq 6 \\ x - 18; & x > 6 \end{cases}$
 Draw the graph of $g(x)$ and discuss the continuity and differentiability of $g(x)$.

6. If $f(x) = \begin{cases} -x, & x \leq 1 \\ 3+x, & x > 1 \end{cases}$ and $g(x) = \begin{cases} 3x, & x \leq 1 \\ 2+x, & x > 1 \end{cases}$, then define $f(g(x))$ and also examine its continuity.

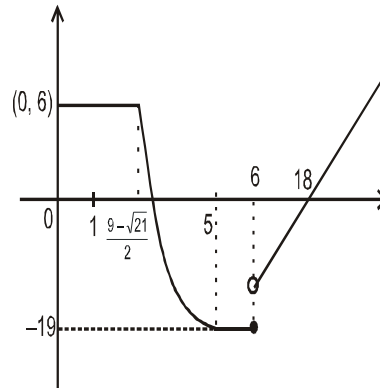
7. If $\cos^{-1}(y/a) = \log(x/n)^n$ satisfies the equation $x^2 \frac{d^4y}{dx^4} + 5x \frac{d^3y}{dx^3} + 8 \frac{d^2y}{dx^2} = 0$, then find the value of n .

8. The distance between the two parallel lines is 1 unit. A point 'A' is chosen to lie between the lines at a distance 'd' from one of them. Triangle ABC is equilateral with B on one line and C on the other parallel line. Find the length of the side of the equilateral triangle

Answers Key

1. (C) 2. (B) 3. (D) 4. (B)(D)
5. $f(x)$ is discontinuous at $x = 6$ and non-differentiable

at $x = \frac{9 - \sqrt{21}}{2}, 6$



6.
$$\begin{cases} -3x, & x \leq \frac{1}{3} \\ 3+3x, & \frac{1}{3} < x \leq 1, \text{ discontinuous at } x = \frac{1}{3} \\ 5+x, & x > 1 \end{cases}$$

7. $n = \pm 2$ 8. $2\sqrt{\frac{d^2 - d + 1}{3}}$

